Fourier dimensionality reduction of radio-interferometric data

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***Abstract*—Next-generation radio-interferometers face a computing challenge with respect to the imaging techniques that can be applied in the big data setting in which they are designed. Dimensionality reduction can thus provide essential savings of computing resources, allowing imaging methods to scale with data. The work presented here approaches dimensionality reduction from a compressed sensing theory perspective, and links to its role in convex optimization-based imaging algorithms. We describe a novel linear dimensionality reduction technique consisting of a linear embedding to the space spanned by the left singular vectors of the measurement operator. A subsequent approximation of this embedding is shown to be practically implemented through a weighted subsampled Fourier transform of the dirty image. Preliminary results on simulated data with realistic coverages suggest that this approach provides significant reduction of data dimension to well below image size, while achieving comparable image quality to that obtained from the complete data set.**

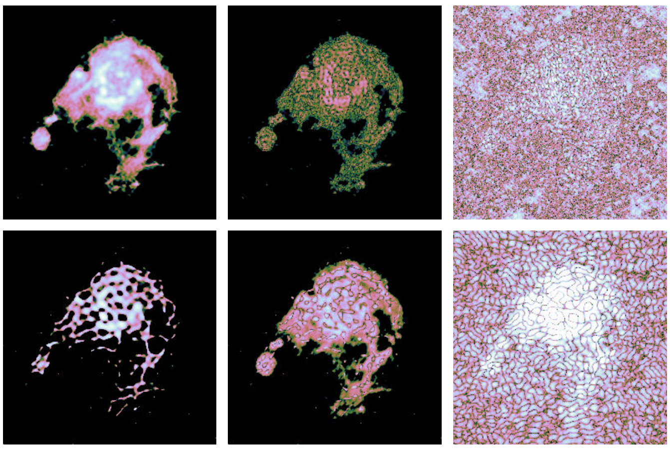
The large amount of data produced from next-generation telescopes like the Square Kilometre Array (SKA) presents a computational challenge for imaging methods, and calls for High Performance Computing (HPC)-ready solutions. Here we present our dimensionality reduction technique as a way to handle big data, and show that radio-interferometric (RI) imaging algorithms applied on significantly reduced data using the proposed method retain image reconstruction quality. The results detailed here are based on preliminary studies as presented in [1].

RI data acquisition can be modelled through the discretized form of a measurement equation, given by $\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{n}$. Here $\boldsymbol{y} \in \mathbb{C}^M$ is a vector of continuous Fourier measurements (visibilities) corrupted by additive noise $\boldsymbol{n} \in \mathbb{C}^M$; we assume $\boldsymbol{n}$ to have i.i.d. Gaussian noise statistics. The visibilities $\boldsymbol{y}$ measure an underlying vectorized image $\boldsymbol{x} \in \mathbb{C}^N$, and $\boldsymbol{\Phi} \in \mathbb{C}^{M\times N}$ is the measurement operator, with $M \gg N$.

Linear dimensionality reduction is performed through an embedding matrix $\mathbf{R} \in \mathbb{C}^{M_L\times M}$, $M_L \ll M$, leading to the reduced inverse problem $\boldsymbol{y}' = \boldsymbol{\Phi}'\boldsymbol{x} + \boldsymbol{n}'$, with $\boldsymbol{y}' = \mathbf{R}\boldsymbol{y}$, $\boldsymbol{n}' = \mathbf{R}\boldsymbol{n}$, and $\boldsymbol{\Phi}' = \mathbf{R}\boldsymbol{\Phi}$. Consequently, imaging algorithms need only deal with the embedded measurement vector of dimension M_L, thus avoiding expensive computations involving large vectors of size M. As an embedding operator, $\mathbf{R}$ affects not only the mapping to $\boldsymbol{y}'$ but also the properties of $\boldsymbol{\Phi}$ required by CS theory to guarantee stable signal recovery. Additionally, retaining the i.i.d. Gaussian properties of the original measurement noise is important for the convex optimization-based algorithms used for image reconstruction.

The optimal dimensionality reduction $\mathbf{R}_{\text{optim}}$, with respect to CS-based image reconstruction, is a projection to the left singular vectors of the measurement operator $\boldsymbol{\Phi}$ that correspond to non-zero singular values. For a Singular Value Decomposition (SVD) of $\boldsymbol{\Phi}$ given by $\boldsymbol{\Phi} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\dagger$, the optimal embedding is then given by $\mathbf{R}_{\text{optim}} = \mathbf{U}_0{}^\dagger = \boldsymbol{\Sigma}_0{}^{-1}\mathbf{V}_0{}^\dagger\boldsymbol{\Phi}^\dagger$, where the final data size $M_L = N_0 \le N$ is the number of non-zero (or *significant*) singular values of $\boldsymbol{\Phi}$, and where $\mathbf{U}_0$, $\boldsymbol{\Sigma}_0$ and $\mathbf{V}_0$ are truncated versions of $\mathbf{U}$, $\boldsymbol{\Sigma}$, $\mathbf{V}$ by only retaining columns (rows for $\mathbf{V}$) corresponding to the N_0 non-zero (or significant) singular values of $\boldsymbol{\Phi}$. However, this requires the SVD, which is computationally expensive and hence infeasible. So, a practical implementation of this optimal embedding is constructed through the approximations $\mathbf{V}^\dagger \approx \mathbf{F}$ and $\boldsymbol{\Sigma}^2 \approx \text{Diag}(\mathbf{F}\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\mathbf{F}^\dagger)$, leading to the embedding operator $\mathbf{R}_{\text{sing}} = \boldsymbol{\Sigma}_0{}^{-1}\mathbf{S}\mathbf{F}\boldsymbol{\Phi}^\dagger \in \mathbb{C}^{N_0\times M}$. In words, this involves the following operations in sequence: computing the dirty image by applying $\boldsymbol{\Phi}^\dagger$, an N-sized Fourier transform $\mathbf{F}$, a subsampling through $\mathbf{S}$, retaining only those dimensions corresponding to non-zero (or significant) singular values of $\boldsymbol{\Phi}$, and finally, a weighting $\boldsymbol{\Sigma}_0{}^{-1}$. The weighting ensures that the noise covariance matrix in the embedded dimension has diagonal elements corresponding to the original variance of the measurement noise $\boldsymbol{n}$. We also note that $\mathbf{R}_{\text{sing}}$ has a fast implementation as it consists of diagonal, sparse and Fourier matrices only. Simulations were performed to compare this proposed dimensionality reduction $\mathbf{R}_{\text{sing}}$ with a weighted subsampled version of the standard 'gridding' operation $\mathbf{G}$ performed in radio interferometry, given by $\mathbf{R}_{\text{grid}} = \overline{\mathbf{WSG}} \in \mathbb{C}^{\overline{N}\times M}$ ($\overline{N} \le 4N$ for an oversampling factor of 2 in the computation of the Fourier transform).

Here we show reconstruction results on an $N = 256 \times 256$ model image of the M31 Galaxy. $M = 50N$ continuous visibilities are sampled following a realistic SKA-like uv coverage. The 'input' SNR, defined as $\text{ISNR} = 20\log_{10}(\|\boldsymbol{y}_0\|_2/\|\boldsymbol{n}\|_2)$ with $\boldsymbol{y}_0 = \boldsymbol{\Phi}\boldsymbol{x}$ being visibilities uncorrupted by noise, is set to 30 dB. Similarly, the 'output' SNR is defined as OSNR $= 20\log_{10}(\|\boldsymbol{x}\|_2/\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2)$, $\hat{\boldsymbol{x}}$ being the reconstructed image. Our simulations show that an OSNR of ≈25 dB is reached in the absence of embedding, although at a heavy computational cost owing to the 3.2 million visibilities. Reconstruction after $\mathbf{R}_{\text{grid}}$ achieves an OSNR of ≈25 dB with $M_L = 4N$. Crucially, much more aggressive dimensionality reduction is possible with $\mathbf{R}_{\text{sing}}$, which obtains an OSNR of ≈24.5 dB, but from a data size $M_L = 0.25N$. The robustness of $\mathbf{R}_{\text{sing}}$ compared to $\mathbf{R}_{\text{grid}}$ is illustrated in the figure below through the reconstructed, error and residual images for both methods embedding data to 5% of image size. We note that image reconstruction from data embedded to $0.05N$ using $\mathbf{R}_{\text{grid}}$ is poor compared to $\mathbf{R}_{\text{sing}}$, as is seen from the artefacts in the reconstructed image and the more prominent residual structure in the bottom row for $\mathbf{R}_{\text{grid}}$.



References

[1] S. V. Kartik, R. E. Carrillo, J.-P. Thiran, and Y. Wiaux, "A fourier dimensionality reduction model for big data interferometric imaging," *arXiv:1609.02097 [astro-ph]*, Sep. 2016.